Project 4

Vorticity-Stream Function Method

Due: Mon., Dec. 6, 2010 at 6:00 pm

Consider the incompressible laminar flow in the plane channel shown below. The dimensions and the boundary conditions (non-dimensionalized) are as shown on the figure. The flow is governed by vorticity $\xi(x,y,t)$ and stream function $\psi(x,y,t)$ transport equations (non-dimensionalized):

$$\frac{\partial \xi}{\partial t} + \frac{\partial u\xi}{\partial x} + \frac{\partial v\xi}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\xi$$

where $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The Reynolds number $Re = 10$, based on the channel height h. Use an explicit FTCS scheme to obtain the *steady-state* solution of these equations. Take $N_x = 50$ and $N_y = 20$ grid points along x and y, respectively. Take the initial condition similar to the inlet boundary condition. The time step Δt satisfies the stability criteria: $\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) \le \frac{1}{2}Re$ and $\frac{|u|_{max}\Delta t}{\Delta x} \le 1$. For your report:

1. Show the steady-state u-velocity profile along the channel.
2. Compare the computed steady-state fully-developed u-velocity profile with the analytical solution given by: $u = 6(y - y^2)$.
3. Visually inspect the evolution of velocity profile from question 1. Is the entrance length predicted by CFD, consistent with the classical formula: $L_e/h \approx 0.06Re$?