# Project 3

## Elliptic PDEs

*Due: Mon., Nov. 22, 2010 at 6:00 pm*

Suppose that it is required to obtain the steady-state temperature distribution on a two-dimensional rectangular plate. The governing PDE for this problem is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

The plate has dimensions of $L = 0.01\,m$ and $h = 0.02\,m$ in $x$ and $y$ directions, respectively. The temperature at the boundaries are specified as:

$$T(0, y) = 0\ ;\ T(x, h) = 0\ ;\ T(x, 0) = T_0 = 100\,K\ ;\ \frac{\partial T}{\partial x}(L, y) = 0 \tag{2}$$

Use a 5-point formula (second order accurate) finite-difference approximation along with second order accurate boundary conditions. Solve the resulting system of equations using:

1. PSOR
2. LSOR
3. ADI

The analytical solution is:

$$T_a(x, y) = \sum_{n=1}^{\infty} \frac{2T_0}{n\pi(1 - e^{\frac{-n\pi h}{L}})} \left(1 - cos(n\pi)\right) \left[e^{\frac{-n\pi y}{2L}} - e^{\frac{-n\pi h}{L}} e^{\frac{n\pi y}{2L}}\right] sin(\frac{n\pi x}{2L}) \tag{3}$$

Discuss the following:

1. Plot the residual vs. number of iteration for each method. Use different relaxation factors for PSOR and LSOR.
2. What relaxation factor values make PSOR and LSOR methods to converge faster?
3. Compare the CPU times of these methods.
4. Compare the numerical results with the analytical solution ($T(x, y)$ vs. $x$) at $y = 0.005, 0.01, 0.015\,m$.