Project 2

Convection-Diffusion PDE

Due: Mon., Nov. 1, 2010 at 6:00 pm

Solve the convection-diffusion equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}$$

describing the wave propagation inside a one-dimensional channel. In this equation:

$$a = 2.5\,m/s\ ;\ \alpha = 0.005\,m^2/s\ ;\ final\ time = 0.2\,s\ ;\ 0 \le x \le 1\,m$$

and it is subject to the initial condition:

$$u(x,0) = \begin{cases} 1.0 & \text{if } x \le 0.2 \\ 0.5 & \text{if } x = 0.2 \\ 0.0 & \text{if } x \ge 0.2 \end{cases}$$

and the boundary conditions:

$$u(0,t) = 1.0\,m/s\ ;\ u(1,t) = 0.$$

The analytical solution is obtained as:

$$u(x,t) = 1 - \frac{1}{2}\left[1 + erf\left(\frac{x - x_0 - at}{2\sqrt{\alpha t}}\right)\right] \quad ; \quad x_0 = 0.2\,m$$

Use the following *explicit* schemes:

1. Finite-Volume: FTCS for both convection and diffusion
2. Finite-Volume: First order upwind for convection, FTCS for diffusion
3. Finite-Difference: FTCS for both convection and diffusion
4. Finite-Difference: First order upwind for convection, FTCS for diffusion
5. Finite-Difference: Lax-Wendroff for convection, FTCS for diffusion
6. Finite-Difference: MacCormack for convection, FTCS for diffusion

Objectives:

1. Compare the solution from different schemes at different times.

2. For at least one of the schemes check the accuracy by increasing the resolution (*i.e.* decreasing Δt, Δx).

3. Discuss: which scheme is more accurate? which ones are more dissipative or dispersive?

Hint: Check the stability criteria, the truncation error and the modified equation of each scheme to explain its behavior for different Δx and Δt values.